Electricity and Magnetism
Lecture -3-

Electric Potential

Electric Potential

- Potential: is potential energy per unit charge.

The potential V defined at any point in an electric field as the potential energy U per unit charge associated with a test charge q_o at that point:-

$$V = \frac{U}{q_o} \quad \text{or} \quad U = q_o V$$

Potential energy and charge are both scalars, so potential is a scalar . The unit of potential called volt , which is:

1 volt = 1 J/C = 1 joule /coulomb.

- Potential energy is the work done by electric force , $W_{a\rightarrow b}$= -Δ U during a displacement from a to b to the quantity - ΔU= - (U_b- U_a), on a " work per unit charge " basis . We divide this equation by $q_{\circ}$, obtaining :-

$$\frac{W_{a\rightarrow b}}{q_{\circ}} = - \frac{\Delta U}{q_{\circ}} = -\left(\frac{U_b}{q_{\circ}} - \frac{U_a}{q_{\circ}}\right)$$

$$= - (V_b - V_a) = V_a - V_b$$

$\therefore\ V_a = \frac{U_a}{q_{\circ}}$ is the potential energy per unit charge at point a , and similarly for V_b .

- Thus the work done per unit charge by the electric force when a charged body moves from a to b is equal to the potential at a minus the potential at b.

  So the work done by the electric force is given by :-

$$W_{a\to b} = \int_{r_a}^{r_b} F\, dr = F\,.r\Big|_a^b = \Delta U$$

$$U_b - U_a = \frac{1}{4\pi\epsilon_o}\frac{q\,q_o}{r_b} - \frac{1}{4\pi\epsilon_o}\frac{q\,q_o}{r_a}$$

where $U_a = \frac{1}{4\pi\epsilon_o}\frac{q\,q_o}{r_a}$ when q_o is a distance r_a from q.

and $U_b = \frac{1}{4\pi\epsilon_o}\frac{q\,q_o}{r_b}$ when q_o is a distance r_b from q.

Thus the potential energy U when the test charge q_o is at any distance r from charge q is :

$$U = \frac{1}{4\pi\epsilon_o}\frac{q\,q_o}{r} \quad \dots \quad \dots .(1)$$

Calculating Electric Potential

- To find the potential V due to a single point charge q , we divide eq. (1) by q_o :-

$$V = \frac{U}{q_o} = \frac{1}{4\pi\epsilon_o}\frac{q}{r} \quad ......(2)$$ [potential due to a point charge]

where r is the distance from the point charge q to the point at which the potential is evaluated.

- If q is positive , the potential that it produces is positive at all points.
- If q is negative , it produces a potential that is negative.
- In either case , V is equal to zero at $r = \infty$

- To find the potential due to a collection of point charges:

$$V = \frac{U}{q_o} = \frac{1}{4\pi\epsilon_o}\sum_i \frac{q_i}{r_i} \quad \dots\dots(3)$$

When we have a continuous distribution of charge along a line, over a surface , or through a volume , we divide the charge into elements dq , and the in eq. (3) becomes an integral :-

$$V = \frac{1}{4\pi\epsilon_o}\int \frac{dq}{r} \quad \dots\dots(4)$$

Finding Electric Potential from Electric Field:-

Can we determine V from electric field $\overrightarrow{E}$. The force $\vec{F}$ on a test charge q_o can be written as :-

$$\overrightarrow{F} = q_o \overrightarrow{E}$$

And we can write the work done by the electric force as the test charge moves from <u>a</u> to <u>b</u> is given by:-

$$W_{a\to b} = \int_a^b \overrightarrow{F} \, . dl = \int_a^b q_o \overrightarrow{E} \, . dl$$

If we divide this eq. by q_o we obtain :-

$$\frac{W_{a\to b}}{q_o} = \int_a^b \overrightarrow{E} \, . dl$$

But $\frac{W_{a\to b}}{q_o} = V_a - V_b$

$$\therefore V_a - V_b = \int_a^b \overrightarrow{E} \, . dl$$

$V_a - V_b = \int_a^b E \cos\theta dl$ ....(1) Potential difference as an integral of $\vec{E}$

If the line integral $\int_a^b \overrightarrow{E}.dl$ is positive, the electric field does positive work on a positive test as it moves from a to b .

The value of $V_a - V_b$ is independent of the path taken a to b , or

$$V_a - V_b = -\int_b^a \overrightarrow{E}.dl \;\ldots\ldots(2)$$

This has a negative sign compared to the integral in eq.(1) and the limits are reversed, hence eqs(1&2) are equivalent .

- If the moving in the direction of $\vec{E}$ ⟶ electric potential V decreases, if you move in direction.

Opposite $\vec{E}$ ⟶ V increase

(a) A positive point charge **(b) A negative point charge**

The unit of potential difference (1 V) is equal to the unit of electric field (1 N/c) multiplid by the unit of distance (1m).

Electron Volts:-

When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the charge in the potential energy U is :-

U_a- U_b= q ($V_a - V_b$)= q V_{ab}

If the charge q equal the magnitude e of the electron charge q, 1.602 x10^{-19} C , and the potential difference is V_{ab}= 1V , the change in energy is :-

U_a- U_b=(1.602 x10^{-19} C) (1V)= 1.602 x10^{-19} Joule

This quantity of energy is defined to be 1 electron volt (1 ev):

1 eV= 1.602 x10^{-19} J

Equipotential Surfaces:-

1. The equipotential surfaces is on at all the point of which the potential has the same value.

2. In equipotential surfaces no work is need to move the charge body over such surfaces.

3. The equipotential surfaces must be at right angle to the direction of the field at any point.

Potential Gradient :

The relationship between electric field and potential :-

$$V_a - V_b = \int_a^b \overrightarrow{E}.dl \quad ....(1)$$

In eq.(1), $V_a - V_b$ is the potential of a with respect to b ,the change of potential encountered on a trip from b to a. we can write this is :-

$$V_a - V_b = \int_b^a dV = -\int_a^b dV \quad .....(1)$$

Where dV is the infinitesimal change of potential accompanying an infinitesimal element dl of the path from b to a. comparing to eq .(1) we have.

$$-\int_a^b dV = \int_a^b \overrightarrow{E}.dl \quad ..... (2)$$

These two integrals must be equal for any pair of limits a and b , and for this to be true . The integrals must be equal . Thus , for any infinitesimal displacement dl ,

$$-dV = \vec{E}.dl \quad ......(3)$$

To interpret this expression , we write $\vec{E}$ and dl in terms of their components:-

$\vec{E}$ = i Ex + j Ey + k Ez

$\overrightarrow{dl}$ = i dx + j dy + k dz ; then we have :-

- dv = $\vec{E}$. $\overrightarrow{dl}$ = Ex dx + Ey dy + Ez dz .... (4)

- dv = Ex dx   Ex = - $\frac{dv}{dx}$

- dv = Ey dy   Ey = - $\frac{dv}{dy}$

- dv = Ez dz   Ez = - $\frac{dv}{dz}$

Or

Ex = - $\frac{dv}{dx}$, Ey = - $\frac{dv}{dy}$, Ez= - $\frac{dv}{dz}$

We can write $\vec{E}$ as

$$\vec{E} = - \left(i\frac{dv}{dx} + j\frac{dv}{dy} + k\frac{dv}{dz} \right)$$

In vector notation the following operation is called the gradient of the funcion f :-

$$\vec{\nabla} f = - \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz} \right) f$$

The operatoor denoted by the symmbol $\vec{\nabla}$is called (grad) or (del) thus in vector notation :-

$$\vec{E} = - \vec{\nabla} V$$

The quantity $\vec{\nabla}$ V is called the ″Potential gradient”

Potential at a point a long the axis of a dipole :-
(and electric field)

$$V = \frac{1}{4\pi\epsilon_o}.\frac{q}{r}$$

$$= \frac{1}{4\pi\epsilon_o}[- \frac{q}{r+l/2} + \frac{q}{r-l/2}]$$

$$= \frac{q}{4\pi\in_o}[\frac{1}{r-l/2} - \frac{1}{r+l/2}]$$

$$= \frac{q}{4\pi\in_o}[\frac{r+\frac{l}{2} - r + \frac{l}{2}}{r^2 - l^2/4}]$$

$$= \frac{ql}{4\pi\in_o}[\frac{1}{r^2 - l^2/4}]$$

if r>> l $\quad l^2/4$ neg.

Moment P = $q \; l$

$\therefore V = \frac{P}{4\pi\epsilon_o r^2}$ potential at a point a long axis of dipole

*Potential at a point on the perpendicular bisector of the dipole:-

$$V = \frac{1}{4\pi\epsilon_o} . \frac{q}{r}$$

$$V = \frac{1}{4\pi\epsilon_o} \left[\frac{q}{s} - \frac{q}{s}\right]$$

$$\therefore V = zero$$

*The potential at a point on the axis of (uniformly charged). (and electric field)

$$V = \frac{1}{4\pi\epsilon_o}.\frac{dq}{s}$$

$$s^2 = a^2 + b^2$$

$$\therefore s = \sqrt{a^2 + b^2}$$

$$V = \frac{1}{4\pi\epsilon_o}\int\frac{dq}{s}$$

$$= \frac{1}{4\pi\epsilon_o}\frac{q}{s}$$

$$V = \frac{1}{4\pi\epsilon_o}.\frac{dq}{\sqrt{a^2+b^2}}$$

*Potential for charged sphere (uniformly charged)

Suppose the charge is q sphere radius a.

1) At point outside the sphere (p) :-

$$E_p = V = \frac{1}{4\pi\epsilon_o}.\frac{q}{r^2}$$

$$\therefore V_p = \frac{1}{4\pi\epsilon_o}.\frac{q}{r}$$

2) At a point placed on the axis for sphere (R):-

$$E_R = \frac{1}{4\pi\epsilon_o} . \frac{q}{a^2}$$

$$\therefore V_R = \frac{1}{4\pi\epsilon_o} . \frac{q}{a}$$

3) At inside sphere (or in center of sphere) at point Q:-

E_Q=Zero

$\therefore V_Q$=Zero